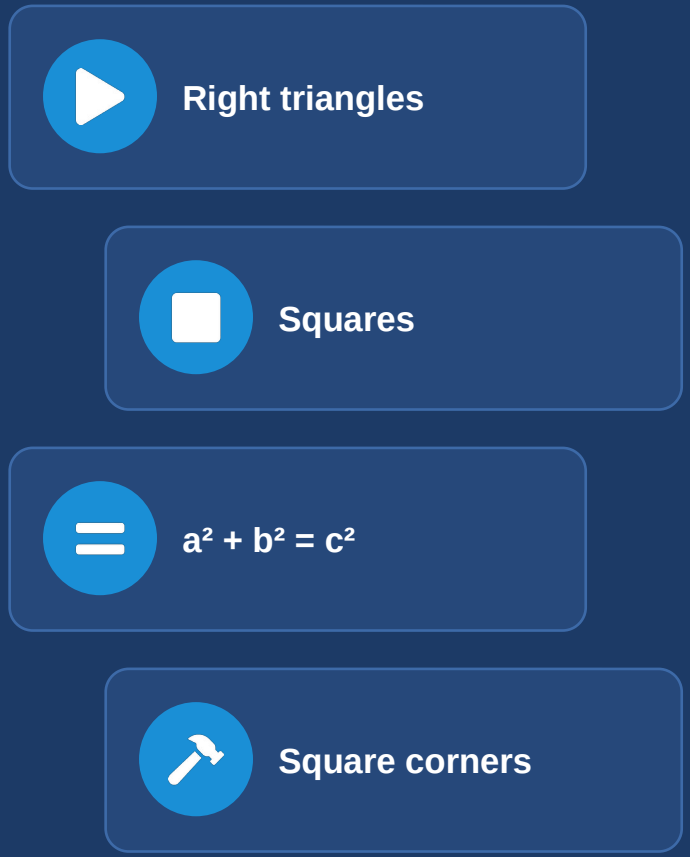
MTH1W Geometry and Measurement, Lesson 6

The Pythagorean Relationship

$a^2 + b^2 = c^2$



Today's goals

We are learning…

- The side-length relationship for right triangles.
- To use it to find missing sides and check right angles.

I can…

- Identify the hypotenuse and legs.
- Find a missing hypotenuse.
- Find a missing leg.
- Test whether a triangle is right-angled.

Today's plan

Time	What we're doing	You will…
0–10 min	**Hook**	Discuss
10–25 min	**Discover**	Investigate
25–40 min	**Use it**	Practise
40–65 min	**Find the side**	Seven problems and a corner check
65–75 min	**Quick check and exit**	Respond

Warm-up

A builder needs a perfectly square corner for a deck, but has no big set square, only a tape measure. How could she check the corner?

Sentence starters

“She could measure…”

“If it's square, then…”

“The diagonal…”

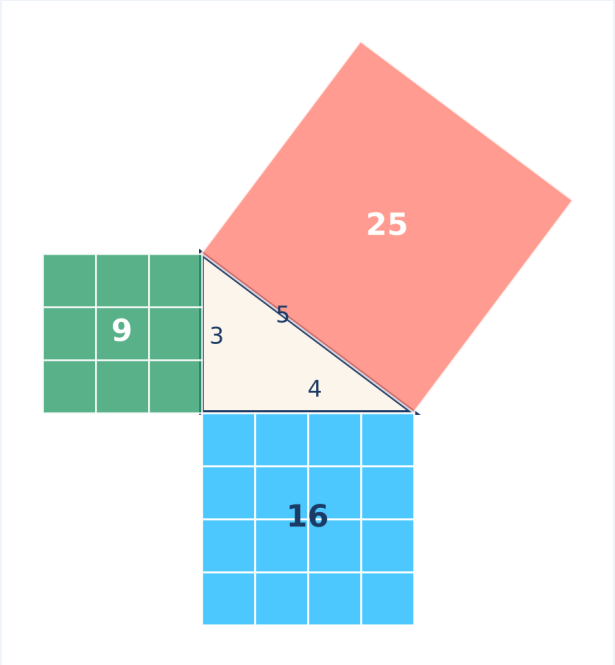
How it works

- Pairs: 3 minutes.

Big idea

In every right triangle, the area of the square on the hypotenuse equals the sum of the areas of the squares on the other two sides: $a^2 + b^2 = c^2$. It lets us find a missing side, and it works backwards too: if $a^2 + b^2 = c^2$, the triangle has a right angle, which builders use to check corners.

Squares on the sides



- Squares on the legs: 9 and 16.
- Square on the hypotenuse: 25.
- 9 + 16 = 25.
- Does it work for every right triangle?

Words to know

legs	The two sides that form the right angle. *a and b*
hypotenuse	The side opposite the right angle; the longest side. *c*
Pythagorean relationship	$a^2 + b^2 = c^2$ in every right triangle. *$3^2 + 4^2 = 5^2$*
converse	If $a^2 + b^2 = c^2$, the triangle is right-angled. *Used to check corners*

Model: finding a leg

Task: A right triangle has hypotenuse 15 and one leg 9. Find the other leg.

$a^2 + b^2 = c^2$
$9^2 + b^2 = 15^2$
$81 + b^2 = 225$
$b^2 = 225 - 81 = 144$
$b = \sqrt{144} = 12$
Check: $9^2 + 12^2 = 81 + 144 = 225 = 15^2$. The leg is shorter than the hypotenuse, as it should be.

Why it works

Puts the hypotenuse as c.

Subtracts to find a leg.

Takes the square root.

Checks.

Try these together

- Legs 9 and 12. Hypotenuse?
- Legs 1 and 1. Hypotenuse (exact and rounded)?
- Hypotenuse 25, leg 7. Other leg?
- Is 4, 5, 6 a right triangle?

Independent: page 2

On your own:

1. Label the hypotenuse c.
2. Hypotenuse missing: add the squares.
3. Leg missing: subtract the squares.
4. Testing a triangle: compare $a^2 + b^2$ with c^2.

The carpenter's check

3-4-5 rule

- Mark 3 units on one side, 4 on the other.
- The diagonal should be exactly 5.
- Scale it: 60, 80, 100 cm.

Why it works

- $3^2 + 4^2 = 5^2$
- The converse: if $a^2 + b^2 = c^2$, it's a right angle.
- No set square needed.

Back to the warm-up: this is how the builder checks her deck.

Quick check

- Legs 9 and 40. Hypotenuse?
- Hypotenuse 13, leg 5. Other leg?
- Is 10, 24, 26 a right triangle?

Check yourself

Be honest. This helps you choose what to work on next.

Success criteria	Not yet	Almost	Got it
I can identify the hypotenuse and legs.			
I can find a missing hypotenuse.			
I can find a missing leg.			
I can test for a right angle.			

Exit ticket

1 Legs 2.5 m and 6 m. Hypotenuse?

2 Explain how a builder checks a square corner.

3 Which side is always the longest in a right triangle?

Next class: Pythagoras in Composite Shapes.

Teacher reveal

Answers: pages 1 and 2

Discover: 3-4: 9 + 16 = 25. 6-8: 36 + 64 = 100. 5-12: 25 + 144 = 169. In each right triangle the two smaller squares add to the largest. Obtuse 3-4 triangle: $c^2 > 25$, so $a^2 + b^2 < c^2$: no match.

In my own words: In a right triangle, the square on the longest side equals the two other squares added together.

1. Legs 6 and 8: $6^2 + 8^2 = 100$: $c = 10$

2. Legs 5 and 12: $25 + 144 = 169$: $c = 13$

3. Legs 7 and 9: $49 + 81 = 130$: $c = \sqrt{130} \approx 11.4$

4. Hypotenuse 15, leg 9: $225 - 81 = 144$: $b = 12$

5. Hypotenuse 10, leg 4: $100 - 16 = 84$: $b = \sqrt{84} \approx 9.2$

6. Sides 8, 15, 17: right triangle: $64 + 225 = 289 = 17^2$: yes

7. Sides 6, 7, 9: right triangle: $36 + 49 = 85 \neq 81$: no

8. Carpenter: $60^2 + 80^2 = 3600 + 6400 = 10\,000 = 100^2$. The diagonal should be 100 cm; 99 cm means the corner isn't square (slightly less than 90°).